LIKELIHOOD RATIO DETECTION OF SIGNALS ON REVERBERATION NOISE

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ABSTRACT

In the context of stochastic dependency of the noise and signal - as in the case of the reverberation noise - the classical methods of signal detection are not applicable. The paper presents a likelihood ratio detection method based on stochastic calculus. The novel aspects of the method are: the signal and the noise need not be stochastically independent and they both can be nonstationary and non-Gaussian. This method is applicable to the active sonar which is a reverberation limited system. Simulations related to the underwater propagation of the acoustic wave show a good performance of the detector at the *receiver operating characteristic* level. The likelihood ratio detection method can also be successfully applied in mobile communication for the channel dynamical allocation, as well as in radar applications.

1. INTRODUCTION

Signal detection theory appeared in the 1940's motivated by the context of the war efforts [1]. Its foundations are strongly connected with the Norbert Wiener's work for a MIT project trying to predict the track of an airplane, as well as the publication of the first book on radar detection [2]. The idea, new at the time, was that the communication of information is a statistical problem and that the performance limits could be calculated from optimization criteria and a systematic approximation designed.

It was acknowledged that the "signal-to-noise ratio" used by the matched filter was not the natural criterion for signal detection. Mark Kaç provided the connection with statistical hypothesis testing, noting that the Neyman-Pearson criterion is adequate for radar detection. The basic operation is to compare a likelihood ratio with a threshold, whose value is determined by a certain criterion. If $X(t)$, $S(t)$, and $N(t)$ are stochastic processes describing the received signal, the transmitted signal and the noise, respectively, then the detection problem consists, in terms of statistical hypotheses tests, of choosing between

$$\begin{cases} \mathbf{H}_0: & X(t) = N(t) & 0 \le t \le T \\ \mathbf{H}_1: & X(t) = S(t) + N(t) & 0 \le t \le T. \end{cases} \quad (1)$$

The strategy provided by Neyman-Pearson criterion assigns the detector to the likelihood ratio expressed as a Radon-Nikodym derivative $\frac{dP_{S+N}}{dP_N}$. This is an optimal detector, in the sense that it minimizes the probability of non-detection, for a given probability of false alarm. In particular this fits the case of radar or sonar detection, where it is hard to judge the implications of not detecting a target but the acceptable probability of false alarm can be determined. The performance of this detection method is usually measured by means of the receiver operating characteristic (ROC), obtained by plotting the probability of detection versus the probability of false alarm.

Along with the expansion of application from radar to sonar, remote sensing and pattern recognition, the noise models evolved from white Gaussian noise to coloured Gaussian noise and randomly modulated jump processes. Following these ideas, the likelihood ratio algorithm presented here is applicable, under minimal assumptions, to the detection of a random signal of unknown law, disturbed by a noise with filtered Wiener and Poisson components. Such models, as discussed at length in [3] and [4], are applicable when the noise is very nonstationary and the signal cannot be represented as a set of narrowband components. Typical examples come from the radar and sonar areas [5]. As the considered stochastic processes will be defined not on an arbitrary abstract probability space, but in the space of the sample paths of the received signal, sometimes notation $X(t)$, for example, will be given explicitly in the form $X(f, t)$ where f is a particular sample path of the received signal.

2. THE DETECTION MODEL

The motivation of the detection model presented here arises from active sonar applications.

In the active sonar technique the injected signal - generated by the source with the objective of detecting the presence of a target - is distorted by the underwater channel consisting of surface, bottom and volume scatterers. If the target is present then the signal will "contact" the target, and this contact represents the signal to be detected at the receiver. In addition to the fading, the injected signal is distorted by the echoes due to returns from surface, volume and bottom scatterers. These form an additional component of the noise, called *reverberation noise* [6]. In fig. 1 the fading effect, the reverberation and the background noise at the receiver are shown. The background noise is Gaussian in general, and is stochastically independent with the contact signal. The reverberation noise is non-Gaussian and stochastically dependent with the contact signal, because they contain distortions from the same environment of the same injected signal [6].

The active sonar system is said to be a *reverberation limited environment* because the reverberation component dominates the background noise. Since the background noise exists equally in the presence or absence of the target, the detection model does not consider it.

The signal observed at the receiver is modeled as an oscillation process defined as

$$X(t) = \sum_k \gamma_k e^{iu_k t} \quad (2)$$

where γ_k are random variables. Hence, this is the superposition of oscillations with frequency $u_k/2\pi$.

The reverberation aspect, leads to the assumption that in relation 2 which describes the oscillation process, the random variables $(\gamma_k)_k$ are correlated, i.e. $E(\gamma_k \gamma_j) = g_{kj} < \infty$. Then
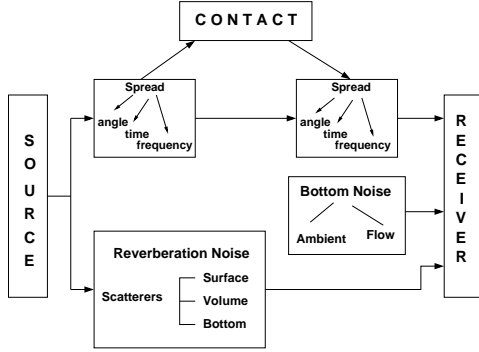
Fig. 1. Active sonar diagram: reverberation limited environment when reverberation noise surpasses the bottom noise.

$X(t)$ is not a stationary process and cannot be studied by means of the linear theory of random processes, as Fourier transforms of an orthogonal stochastic measure [7]; $X(t)$ is therefore a particular case of an *harmonizable process* [8]. In usual situations, the variance of the received signal, $X(t)$ is finite. Then, $X(t)$ can be represented by means of the Cramér-Hida decomposition [9], i.e., the signal can be viewed as a superposition of stochastic integrals with respect to stochastic processes with orthogonal increments $B_k(t), 1 \leq k \leq K$ in the form given by

$$X(t) = \sum_{k=1}^{K} \int_0^t F_k(t, x) dB_k(x) \qquad (3)$$

where $F_k(t, s)$ are causal transforms: $F_k(s, t) = 0$ if $s > t$. K is the *Cramér-Hida multiplicity* of $X(t)$ and gives the complexity of the random process. Computational restrictions lead to a limitation of the model to $K = 1$ and to a subclass of stochastic processes with orthogonal increments, namely the set of processes with independent increments. Lévy decomposition [10] proves that the processes with independent increments are essentially generated by the sum of Gaussian and Poisson processes. Then, the received process is modeled by

$$\begin{aligned} X(t) &= \int_0^t F(t, x) dB(x), \text{ where} \qquad (4) \\ B(t) &= \frac{1}{2} \left[B_1(t) + \tilde{B}_2(t) \right] \end{aligned}$$

with $B_1(t)$ a generalized Brownian motion and $\tilde{B}_2(t)$ a Poisson martingale, both having the same variance function $\beta(t)$. $B(t)$ will be addressed as unfiltered noise. In this form, $X(t)$ models non-message bearing signal or "non-intelligent" noise.

The presence of a target on the channel makes at least one oscillation from the fluctuations modeled by the oscillation process have a particular behaviour: it becomes smoother than the other oscillations, i.e., it has an "intelligent" character [11].

The signal observed at the receiver has therefore one outstanding component in the oscillation process model; this component is modeled by a stochastic process $s(t)$ which includes the informa-

tion carried by the target, in the form

$$X(t) = \int_0^t F(t, x) \left[s(x)\beta(dx) + dB(x) \right]. \qquad (5)$$

The same factor $F(t, x)$ multiplies both the noise and the "signal" $s(t)$ as a consequence of the fact that the injected signal is their common root. The statistical distribution of $s(t)$ is supposed to be unknown.

Hence, the detection problem consists of determining for a given observed signal at the receiver which one of the relation 5 or 4 applies. The channel modeling used for sonar applications does not differ in essence from that used in mobile communications. In addition to the distortion produced by fading, signals on a wireless channel may be affected by interference, a phenomenon for which the uncorrelation assumption is not appropriate. For this situation, as for the case of the reverberation phenomenon, the model proposed here may be used.

3. LIKELIHOOD RATIO DETECTION ALGORITHM

When the detector is chosen to be based on a likelihood ratio, in order to obtain a rigorous solution the following four operations have to be successfully accomplished.

A. Establish the existence of the likelihood ratio. Technically this means that the absolute continuity of P_{S+N} with respect to P_N has to be proved.

B. Derive explicitly the likelihood ratio, when it exists, as a functional Λ, computable for each received signal and without knowing which of the P_{S+N} or P_N regimes are applicable.

C. Determine the threshold Λ_0 required for decision when the functional Λ is available. A Λ_0 is associated with every predefined probability of false alarm δ and can be obtained from the equation

$$\delta = P_N \left(f \in L_2[0, T] : \Lambda(f) > \Lambda_0 \right). \qquad (6)$$

Also, for every Λ_0 the probability of detection $1 - \eta$ is obtained from the relation

$$\eta = P_{S+N} \left(\Lambda(f) \leq \Lambda_0 \right). \qquad (7)$$

D. Find a discretization for which the likelihood ratio satisfies 6 and 7. Since the received signal is usually observed in discrete form, for example $f(t_1), f(t_2), ..., f(t_n)$, it has to be checked that approximations Λ_n of Λ provide

$$\begin{aligned} P_N \left(\Lambda_n \left(f(t_1), \ldots, f(t_n) \right) > \Lambda_0^{(n)} \right) &\approx \delta \\ P_{S+N} \left(\Lambda_n \left(f(t_1), \ldots, f(t_n) \right) > \Lambda_0^{n} \right) &\approx 1 - \eta \end{aligned}$$

where $\Lambda_0^{(n)}$ is the value of the threshold obtained when Λ is replaced by its approximation Λ_n in relation 6 .

While the results of points (C) and (D) in the previous description may be strongly dependent on the particular features of the detection problem, the answers to the points (A) and (B) require a theoretical approach only. For the detection model considered above, the results from [12, Thm. 2] provide the mathematical tools in order to be able to fulfill operations (A) and (B). In this context, the answers to (A) and (B) above are:

$\tilde{A}$. P_{S+N} is absolutely continuous with respect to P_N if the signal's finite energy condition

$$P \left(\int_0^T s^2(x) d\beta(x) < \infty \right) = 1 \qquad (8)$$

holds.

$\tilde{B}$. The existence of the functional Λ having the required properties from (B) above is warranted if, in addition, P_N is absolutely continuous with respect to P_{S+N}. A sufficient condition for that is

$$P_N \left(\int_0^T s^2 (M \circ f, x) d\beta(x) < \infty \right) = 1 \qquad (9)$$

where M is called the *inversion process*[12, 5.2]. It has the property that, for a fixed $t \in [0, T]$, $M(t) \circ N = B(t)$ so it can be thought of as a whitening filter. Its construction is presented in the algorithm below. The condition 9 is satisfied if

$$E \left[\exp \left\{ \frac{1}{2} \int_0^T s^2 (\cdot, x) \beta(dx) \right\} \right] < \infty.$$

Condition 8 is generally satisfied for the common types of signals met in practice. The main steps of the algorithm to perform for the computation of the functional Λ are described below. This algorithm requires knowledge of the unfiltered noise variance $\beta(t)$; the span of time T available for observation;the covariance of the filtered noise or the causal filter transform application $F(t, s)$; modulation parameters of the injected signal.

The received signal is assumed to be a continuous waveform $f(t)$ such that $\int_0^T f^2(x) dx < \infty$.

The algorithm consists of the following steps:

Step 1. Compute (if $F(t, s)$ is known) or estimate (if $F(t, s)$ is unknown) the noise covariance
$C_N(t, \tau) = \int_0^{t \wedge \tau} F(t, x) F(\tau, x) d\beta(x).$

Step 1'. If $F(t, s)$ is unknown compute or estimate it from the noise covariance.

Step 2. Compute the eigenvalues λ_i, $1 \leq m$, and the orthonormal eigenvectors e_i, $1 \leq m$, (m can be finite or infinite) of the covariance operator associated with $C_N(t, \tau)$ (Principal Components).

Step 3. Approximate the inversion process $M(f, t)$ by $M_n(f, t) = \sum_{i=1}^n M^{(i)}(f, t)$ where $n \leq m$,

$$M^{(i)}(f, t) = \frac{1}{\lambda_i} \langle U I_{[0,t]}, e_i \rangle_{L_2[0,1]} \langle f, e_i \rangle_{L_2[0,1]}$$

and $U I_{[0,t]}(\tau) = \int_0^{t \wedge \tau} F(\tau, x) dx$. $I_{[0,t]}$ is the characteristic function of the interval $[0, t]$.

Step 4. Compute the functional $\tilde{\Lambda}$ giving the likelihood ratio for the unfiltered processes,

$$\tilde{\Lambda}(f) = \frac{dP_{\int_0^{\cdot} s(x) d\beta(x) + B(\cdot)}}{dP_B}$$

from

$$\tilde{\Lambda}(f) = \exp \left\{ \int_0^T s(f, x) ev(f, dx) \right.$$
$$- \frac{1}{4} \int_0^T s^2(f, x) \beta(dx)$$
$$\left. - \frac{1}{\sqrt{2}} \int_0^T s(f, x) \tilde{B}_2(f, dx) \right\}$$

where $ev(t)$ denotes the stochastic process defined on the space of sample paths of the received signals by $ev(f, t) = f(t)$.

Step 5. Compute the functional $\Lambda(f) = \frac{dP_{S+N}}{dP_N}(f)$ from

$$\Lambda(f) = \tilde{\Lambda} \circ M(f).$$

4. IMPLEMENTATION

Assume that values of an observed waveform $x(t)$ are available at times $0 = t_0 < t_1 < \cdots < t_{n-1} < t_n = T$ stored into a vector $\boldsymbol{t}_n$. The corresponding values of f are denoted $x(\boldsymbol{t}_n)$. Then, using standard approximations of integrals,

$$\ln \left[\frac{dP_{S+N}}{dP_N} \right](x) \approx \sum_{i=0}^{n-1} s(M(x, t_i)) [M(x, t_{i+1}) - M(x, t_i)]$$
$$- \frac{1}{4} \sum_{i=1}^{n-1} s^2(M(x, t_i)) (t_{i+1} - t_i)$$
$$- \frac{1}{\sqrt{2}} \sum_{i=0}^{n-1} s(M(x, t_i)) \left[\tilde{B}_2(M(x, t_{i+1})) \right.$$
$$\left. - \tilde{B}_2(M(x, t_i)) \right]$$

Let $\Sigma_N(\boldsymbol{t}_n)$ denote the covariance matrix of the noise variables $N(\cdot, t_0), N(\cdot, t_1), \ldots, N(\cdot, t_n)$, and let L_n denotes the matrix that transforms the vector $\boldsymbol{u}$ into a vector $L\boldsymbol{u}$ whose ith component is $[L\boldsymbol{u}]_i = \sum_{k=1}^i u_k$.

$\Sigma_N(\boldsymbol{t}_n)$ can be decomposed as $\Sigma_N(\boldsymbol{t}_n) = F_n F_n^\star$, where F_n is lower triangular. As $M(x, t_i) \approx \left[L_n F_n^{-1} x(\boldsymbol{t}_n) \right]_i$, denoted by y_i, when $[\boldsymbol{t}_n]_{i+1} - [\boldsymbol{t}_n]_i = \tau$, $1 \leq i \leq n-1$, an approximation $\mathcal{L}_n(x, \boldsymbol{t}_n)$ of the logarithm of the likelihood $\ln \left[\frac{dP_{S+N}}{dP_N} \right](x)$ is obtained in the form

$$\mathcal{L}_n(x, \boldsymbol{t}_n) = \sum_{i=0}^{n-1} s(y_i) \left[F_n^{-1} x(\boldsymbol{t}_n) \right]_{i+1} - \frac{\tau}{4} \sum_{i=0}^{n-1} s^2(y_i)$$
$$- \frac{1}{\sqrt{2}} \sum_{i=0}^{n-1} s(y_i) \left[\tilde{B}_2(y_{i+1}) - \tilde{B}_2(y_i) \right]$$

which can be given in a recursive form (when a new observation $x(t_{n+1})$ is registered.The only new term to compute at step $n+1$ is the term $\left[F_{n+1}^{-1} x(t_{n+1}) \right]_{n+1}$ which can be obtained from the cross correlation of $x(t_{n+1})$ with the $(n+1)$th row of F_{n+1}^{-1}.

When the data about the injected signal modulation are not enough, s is estimated from the data. Some details can be found in [13], and comments about practice follow.

5. THE RECEIVER OPERATING CHARACTERISTIC

Based on large training sequences, the likelihood ratio detector presented above was compared to three classical detectors. Their structure is given below, as a function of the covariance matrix of the noise $\boldsymbol{R}_N$, covariance function of the signal-plus-noise $\boldsymbol{R}_{S+N}$, the mean vector of the noise m_N and of the signal-plus-noise m_{S+N}.

1. Gaussian versus Gaussian (GvG) detector is known as optimal when the signal to be detected and the noise are Gaussian random processes [14]. The detector is given by

$$\Lambda_1(x) = (x - m_N) \boldsymbol{R}_N^{-1} (x - m_N)^*$$
$$- (x - m_N) \boldsymbol{R}_{S+N}^{-1} (x - m_N)^*$$
$$+ 2(x - m_N)^* \boldsymbol{R}_{S+N}^{-1} (m_{S+N} - m_N)^*.$$

2. Differential likelihood detector(DFL) requires also the knowledge of the second order statistics:

$$\Lambda_2(x) = (x - m_N) \boldsymbol{R}_N^{-1} (\boldsymbol{R}_{S+N} - \boldsymbol{R}_N)) \boldsymbol{R}_N^{-1}$$
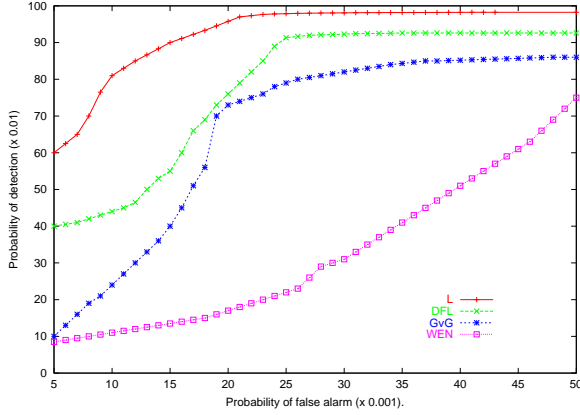$$(x - m_N)^* + 2(x - m_N)^* \boldsymbol{R}_N^{-1} (m_{S+N} - m_N)^*.$$

Fig. 2. Comparison between ROC of the detector given by L and three classical different detectors.

3. Whitening energy detector (WEN) depends only on the second order statistics of the noise:

$$\Lambda_3 = x^* \boldsymbol{R}_N^{-1} x.$$

The receiver operating characteristic curves associated with our likelihood ratio detector $\mathcal{L}$, and GvG, DFL and WEN are presented in the figure 2. $\mathcal{L}$ outperformed, with a probability of detection of 0.98 for a probability of false alarm of 0.02. For the same probability of false alarm, DFL gives 0.82 probability of detection, GvG 0.73 probability of detection while WEN only 0.17.

6. CONCLUSIONS

As can be seen, the theory developed here does not provide, as do approaches such as matched filtering, an algorithm that can be implemented in a straightforward manner for a given system. Rather, it provides a likelihood-ratio-based framework within which an effective implementation can be found. The development of effective algorithms based on the theory is dependent on analysis of data properties and representations. The positive part is the existence of an likelihood ratio based approach allowing to proceed with confidence and which applies to very general signal-plus-noise processes, and without using independence assumptions. An other novelty is that the effect of the communication channel is modeled by the Cramér-Hida framework, as a causal transformation corresponding to the time variant systems arising in real applications.Also, a new feature of the model is the impulsive noise component, represented by a filtered Poisson process, which fits some types of "non-intelligent" noise [11] arising in communication systems as interference which is incoherent relative to the transmitted signal.

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